# Engineering Notes

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## Langmuir Probe Measurements in a Discharge from a Hollow Cathode

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USE of the vapor-fed hollow cathode both as plasma-bridge neutralizer and as the ion chamber electron source has been advantageous in mercury bombardment ion thrusters.[1–3] A hollow cathode differs from earlier types (e.g., refractory metal ribbon, or oxide magazine) in several important ways. For example, it is a source of mercury atoms, as well as of electrons. Consequently, a very large pressure gradient exists just outside the hollow-cathode orifice as a result of the mercury vapor expanding into a region of low pressure. Also, electron injection is from a much smaller area than it is with other types of cathodes. The properties of the external discharge plasma obtained also differ, and it was necessary to introduce a baffle in the discharge chamber of the electron bombardment thruster to obtain good performance when the discharge was fed by a hollow cathode.[3]

To investigate the nature of the hollow-cathode discharge, Langmuir probe measurements were made in a diode configuration with a plane circular anode placed opposite the hollow-cathode. The hollow cathode assembly consisted of a 0.32-cm-outside-diam tantalum tube with a 0.1-cm-thick, 2% thoriated tungsten-alloy disk attached to one end by electron-beam welding. A 0.02-cm-diam orifice was cut in the center of the tungsten disk. A barium carbonate coated tantalum foil insert placed inside the tube facilitated starting the discharge. In operation, the cathode tube was electrically heated by a tungsten-rhenium wire wound around it and embedded in a flame-sprayed coating of alumina.

The anode was a 14-cm-diam stainless steel disk. The distance of the cathode could be varied. The experiments were conducted in a 45-cm-diam vacuum bell jar. The typical pressure during the runs was on the order of $10^{-6}$ torr.

Three modes of operation could be obtained depending on the value of the neutral flow, the cathode to anode distance and the discharge circuit variables. In the highest current mode the discharge was not visible except for an intense bluish white spot just downstream of the orifice, extending less than a few mm. This is referred to as the "spot" mode. The power supply was operated current-limited at 2.0 amp requiring 13.5 v. The "plume" mode was characterized by a visible plume of blue plasma extending forward in a cone several cm from the cathode orifice. The plume mode was operated voltage limited at 35 v which resulted in a current of about 0.3 amp. With very low neutral flows and large anode distances a third mode was found. In this mode, only up to 0.3 amp of discharge current flowed at 60 v. It had the appearance of a low-intensity plume mode.

Figure 1 shows the approximate ranges of anode distance and neutral flow in which the various modes were obtained. A region of overlap is shown between the spot and plume modes; stable operation of either mode is possible over an appreciable range of discharge currents. At or below 20 ma of neutral flow, it was difficult to maintain anything but a high voltage plume mode at medium or long anode distances. At very short distances, however, the spot mode was obtained.

For the Langmuir probe runs, the anode distance was fixed at 3 cm. The neutral flow was between 45 and 55 equivalent ma. Langmuir probe traces were obtained at various probe positions along the symmetry axis of the discharge. In Fig. 2, plasma potential and electron temperature, as determined from probe traces, are plotted vs probe position. The plasma densities ranged from $10^{15}$ to $10^{17}$ electrons/m$^3$ for the plume mode and about an order of magnitude higher for the spot mode. The density dropped rapidly with distance from the cathode, as expected.

### Discussion of Results

Based on an analysis of the data and some theoretical calculations some tentative conclusions were drawn as to the nature of the two principal modes, the spot and the plume modes. The main phenomenological features of the two modes are summarized in Table 1.

A density profile was estimated for a neutral efflux of mercury vapor from the hollow cathode, and this was compared with measured electron densities to obtain the approximate ionization fraction. The comparison indicates that in the spot mode, the degree of ionization may be quite high over the entire discharge region, while in the plume mode it is probably considerably lower.

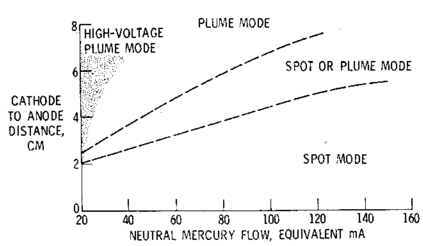


**Fig. 1 Ranges of anode distance and neutral flow for various discharge modes.**

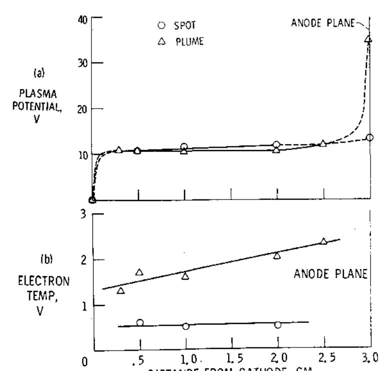


**Fig. 2 Plasma potential and electron temperature distribution in hollow-cathode discharge along symmetry axis. Approximate neutral flow, 50 equiv. ma.**

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**Table 1 Comparison of characteristic properties of spot and plume modes**

| Characteristics typical values | Spot | Plume |
|---|---|---|
| Appearance | Intense spot within a few mm radius from exit hole | Luminous bluish plume-like region extending from cathode to about two-thirds of distance to anode |
| Neutral flow, $J_{Hg}$, equivalent ma | ~50 and higher | ~50 and lower |
| Discharge current, $J_I$, amp | 2.0 | 0.3 |
| Electron to atom ratio, $J_I/J_{Hg}$ | 40 | 6 |
| Discharge voltage, $\Delta V_I$, v | 13.5 | 35 |
| Plasma potential (with respect to cathode),[a] v | 11 | 11 |
| Electron density (0.5 cm from cathode), $n_e$, electrons/m$^3$ | $10^{18}$ | $10^{17}$ |
| Electron temperature,[a] $V_e$, v | 0.5 | 1.0 to 2.3 |
| Estimated ionization fraction, $n_e/n$ | 0.1 to 1.0 | 0.01 to 0.1 |
| Tip temperature (pyrometer), °K | 1600 | 1350 |

[a] For region between cathode and anode.

For both modes the potential was nearly constant over the main body of the discharge (Fig. 2a) at about 11 v above the cathode. This value barely exceeds the first ionization potential of mercury (10.4 v). The edge of this nearly equipotential plateau (where the potential gradient becomes appreciable) will be called the "sheath edge." In these experiments no probe measurements could be made closer than 0.3 cm to the cathode; thus the position of the sheath edge could only be inferred. For a hollow cathode of the type described, it is clear that if the sheath dimension is large compared with the orifice, it will envelop the front surface of the tip, and the geometry of the hole would have little influence on its shape. On the other hand, if the sheath dimension is much smaller than the orifice radius, the sheath may line the contours of the cathode everywhere, including the inside of the orifice channel and the cavity.

A high estimate of the Debye length for conditions that appear to exist inside the hollow cathode in the spot mode is $7 \times 10^{-6}$ m, which is ~3% of the orifice diameter. In the plume mode the estimated Debye length is $\sim 3 \times 10^{-5}$ m, approaching the order of magnitude of the orifice dimensions. This suggests that the spot mode is characterized by the sheath edge deep inside the cavity, while the plume mode has a sheath spanning the orifice on the outside as indicated in Fig. 3.

In the spot mode, according to this model, electrons accelerated through the sheath are projected into the inner region of the cathode. This mechanism explains—at least qualitatively—the higher electron density (thus higher degree of ionization) observed in this mode. Moreover, since most fast electrons produce ionization or excitation and lose their energy inside or very near the tip, the larger portion of the discharge region between cathode and anode is not luminous.

For the plume mode, if the sheath edge is assumed to lie outside the cathode as in the proposed model, it can be shown[4] that only a small portion of the electrons that have been accelerated through the sheath will undergo ionizing collisions. Thus fast electrons are present in the discharge and apparently make enough excitation collisions to give luminousity to the plume.

These investigations indicate then that the two principal modes of operation of the hollow cathode are probably as-

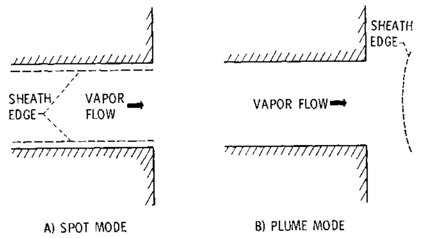


**Fig. 3 Hypothetical sheath-edge positions in orifice channel cross section.**

sociated with different sheath configurations near the cathode. It is hoped that the plasma parameter data presented in this Note will contribute to the understanding of the mechanisms of the discharge in these modes.

### References

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# Linearized Impulsive Earth-Approach Guidance Analysis

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## Introduction

REFERENCES 1–5 discuss the linear theory of impulsive velocity corrections for space vehicle guidance, but the discussions are restricted to fixed-time-of-arrival (FTA) and variable-time-of-arrival (VTA) position guidance, with no attempt to generalize the formulation of the guidance laws. A similarity of form among the guidance laws is evident, which suggests that the laws somehow are related mathematically. Cicolani[6] noted the similarity and attempted to determine the general properties of linearized impulsive guidance laws by using the concepts of linear vector spaces and the pseudoinverse of a matrix. A more straight-forward approach to the problem was made by Tempelman,[7] who started with a generalized linear constraint equation and developed a general solution to the impulsive guidance problem. Ribarich and Meredith[8] have shown that the theory of maxima and minima is another convenient framework for formulating problems in the midcourse guidance area.

The analysis presented in this Note originally was motivated by Tempelman's work and is an attempt to delineate certain types of terminal constraints affecting the particular matrices comprising the general linear impulsive guidance law. A comparison of a variety of guidance laws for the Earth approach phase of a representative conjunction-class Mars mission is presented to illustrate the theory.

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